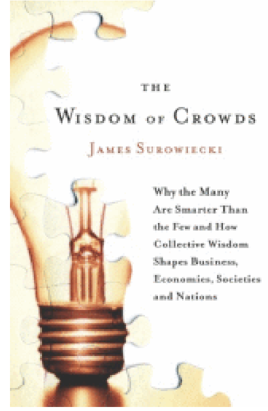
"The Wisdom of Crowds"

Executive Summary



"The Wisdom of Crowds" is quite firmly based on 300 years' work related to Statistical Sampling Theory. James Surowiecki has, not altogether unknowingly, provided a delightful yet very insightful application of Jacob Bernoulli's Law of Large Numbers. Bernoulli's elegant work provided the earliest treatment of a method for estimating the likelihood of the occurrence of a future event. In a very real sense, this was a method used to deal with uncertainty, that is, to estimate or predict the occurrence of an event, not yet observed. In the very simplest of terms, the Law states that the larger the number of uncorrelated observations or estimates of some value regarding an event, the more closely the observations will converge to the true value. Provided the assumptions of the theorem, which are also captured by Surowiecki in his characteristics of "wise crowds", are satisfied, larger numbers of observations will indeed lead to a smaller error in the prediction of an outcome. In a general sense, this is unassailable. Clearly, your wish to improve the accuracy of the estimate of the "Likelihood" of events captured by the Implications Wheel will conform to this theorem provided you attempt to satisfy at least the assumptions of Independence and Diversity.

While the author's treatment of case studies in the second half of the book is at, and sometimes beyond the margin of what the theorem would permit, his fundamental thesis is sound. Statistically, and perhaps practically, the most difficult part of the application of the theorem to the Implications Wheel is to produce diverse and independent estimates. Substantial violations of either of these assumptions will dramatically reduce the aggregated accuracy of the estimates.

I arbitrarily selected two of the works from Surowiecki's bibliographic citations and reviewed his use and interpretation against the original articles. He is accurate in both cases and I detected no bias or unreasonably forced conclusions.

Analytical Evaluation of
"The Wisdom of Crowds"

L. Sharp, September 2004

The principle focus of this evaluation is to provide information on the question of the mathematical and statistical appropriateness of the author's assertion that "under the right circumstances, groups are remarkably intelligent, and are often smarter than the smartest people in them".[1] Various iterations of this major theme are offered - "the crowd's judgment was essentially perfect"; "collective intelligence is often excellent"; "chasing the expert is a mistake" and so on. A more formal statement by the author is as follows: " "If you put together a big enough and diverse enough group of people and ask them to "make decisions affecting matters of general interest", that group's decision will, over time, be "intellectually [superior] to the isolated individual," no matter how smart or well informed he is." " [2] To answer the question of why this is the case, Surowiecki offers a third and more rigorous statement of his thesis: "At heart, the answer rests on a mathematical truism. If you ask a large enough group of diverse, independent people to make a prediction or estimate a probability, and then average those estimates, the errors each of them makes in coming up with an answer will cancel themselves out. Each person's guess, you might say, has two components: information and error. Subtract the error, and you're left with the information."[3]

While he alludes to a "mathematical truism", I am unable to conclude from his evidence whether he explicitly knows the "truism" and simply chose to exclude its' explanation or whether he trusts the reader to know that his case is based on what is popularly termed the "Law of Averages" (a widely used and often quoted argument but usually misunderstood and virtually always misused). In any case, the "mathematical truism" to which Surowiecki refers, whether or not he knows it, is the Law of Large Numbers, more appropriately termed the "Bernoulli Principle" or "Bernoulli's Law of Large Numbers", first presented to the scientific world in 1713. [4] (In case anyone questions the date, since Bernoulli died in 1705, his nephew discovered many unpublished manuscripts and published many of them in 1713 in the "Ars Conjectandi"). Bernoulli's book is regarded by many as the formal beginning of the mathematical theory of probability. Part 4 of the book contains the elegant work on Bernoulli's analysis of the **interpretation of evidence** and this is the material which is fundamental to the application of Surowiecki's work to the Implications Wheel. I trust the following will provide you a measure of comfort and confidence as you develop your link between the Wheel and Crowds.

Bernoulli began his discussion, culminating in the theorem, by observing that, in games of chance, such a rolling a die; the apriori (that is the determination made **before** any evidence was observed) determination of probabilities was straightforward and unambiguous. Simply take the ratio of the event considered a success (for example rolling a 7) to the total possible

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[1] Surowiecki, James. The Wisdom of Crowds Doubleday, 2004, p.xiii.
[2] Ibid. p. xvii
[3] Ibid. p. 10.
[4] Bernoulli, Jacob. 1713. *Ars Conjectandi*. Basil: Thurnisiorum. As contained in Stiegler, Stephen M. 1986. *The History of Statistics. The Measurement of Uncertainty before 1900.* The Belknap Press of the Harvard University Press

outcomes (in the case of a die, 6) - therefore 1/6. You can do this apriori, that is, by intuition before the evidence is in - you need not ever roll a die. He then posed the question of the problems presented when one wished to consider such issues as "disease, weather or games of skill where the causes are hidden and (most important for you and the Wheel), **enumeration of equally likely cases impossible**".[5] In rolling a die, the number of equally likely cases is six, known before a die is ever rolled. Bernoulli correctly concluded that it was not possible (he thought it insane to try) to determine the probability of successes for disease, weather etc. in the same fashion as one would for the roll of a die. Why? Again, **because one cannot enumerate the possible outcomes**. So he proposed to determine the probability of a success a posteriori - that is, by observation of the iteration of the process. "For it should be presumed that a particular thing will occur or not occur in the future as many times as it has been observed, in similar circumstances, to have occurred or not occurred in the past."[6] Thru observation of the process, after enough evidence is observed, one could empirically determine the proportion of successes and enumerate the total possible outcomes. (I know that this idea was not original with Bernoulli but cannot now recall the earlier source.) If you did not know that a die had 6 faces, after several hundred rolls, you could quite accurately conclude that it had 6 faces. Even though this idea did not originate with Bernoulli, what was unique however, was his attempt to give formal mathematical treatment to the vague notion that "the greater the accumulation of evidence about the unknown proportion of cases, the closer we are to certain knowledge about the proportion".[7] Bernoulli accepted what his colleagues commonly acknowledged, that **uncertainty decreased as the number of observations increased**. He wrote "For even the most stupid of men, by some instinct of nature, by himself and without any instruction (which is a remarkable thing), is convinced that the more observations have been made, the less danger there is of wandering from one's goal".[8]

With some limitations, Bernoulli was able to prove this theorem. Stated in more obtuse mathematical terminology, a currently acceptable version of the Law of Large Numbers is - "In repeated independent trials, the chance that the percentage of successes differs from the actual probability converges to zero as the number of trials goes to infinity". Now for an important limitation. In Bernoulli's initial excursions, he wished to achieve "moral certainty", which to him represented only 1 chance in 1000 that the observed N independent trials would produce a value different from the actual probability. For an arbitrarily selected, presumed final ratio, he calculated that the number of trials required to achieve his 1/1000 purity was more than 25,000. That is, one would have to iterate the event more than 25,000 times, in order to specify the likely outcome (with no more than a 1/1000 chance of being wrong). Even by today's statistical standards of proof, a 1 in 20 chance of being wrong, the number of observed iterations of the event would be 8400 (again this is for an arbitrarily selected yet unknown ratio) - still quite large and in many cases impractical. Notwithstanding this, Bernoulli's achievement is remarkable in that he developed a mathematical approach to the measurement of uncertainty - that is, to specify, with decreasing error, the probability with which an as yet to be observed outcome is likely to occur**. Iteration of the event is equivalent to one person, with the characteristics Surowiecki asserts, estimating the probability or likelihood of the occurrence of the event, such as selling or buying a stock, or betting on a**

[5] Ibid. p. 65
[6] Bernoulli, J. *Ars Conjectandi*. 173. p. 224.
[7] Stigler. P. 65
[8] Ibid. p. 65

horse race. Bernoulli's law explains why these systems are so accurate. They do so, primarily because "The crowd is holding a nearly complete picture of the world in its' collective brain"[9] and the number of events iterated ranges from the thousands to the millions.

It is therefore, unambiguous and intrinsically defensible, that the larger the group you sample to determine a likelihood value, the closer the groups' Mean L value will converge to the true value. Said another way, 1000 estimates of L are more accurate than 100 estimates of L. The key question is, how close is close enough to the true value? I think this is not a productive question to pursue. Ultimately, the larger the number of observations, the closer the aggregated group estimate approaches the true value.

At least two of Surowiecki's four conditions that characterize "wise crowds" are absolutely, statistically essential. These are **diversity** and **independence**. Within diversity (the characteristic that each individual must have private information concerning the event in question - even if it is "eccentric") you must tap into but not necessarily (perhaps impossible) capture the realm of existing knowledge concerning the event under investigation. (Recall his Challenger/Thiokol example) Independence ensures that the individual assessments of L are maximally uncorrelated; otherwise the errors do not converge to zero. These are process variables within the context of an organization and, I suspect the most that one can do is strive to achieve the two characteristics and press on.

I arbitrarily selected two articles for systematic review, in order that we might have pretty much unconditional confidence in his interpretation and use of the literature he cites. Problem Solving by Heterogeneous Agents, *Journal of Economic Theory*, 97, (2001): 123 - 163 is particularly valuable to his thesis.[10] The Colorado College Library could only obtain a hard copy for $40 so I read it on-line. It is an excellent article and I recommend you try to secure a copy. In case you need to defend your argument regarding use of a large group to estimate L, you could comfortably cite this article and be on very solid ground. In any case, Surowiecki has appropriately used the content of the article and I detect no bias in his presentation. I also reviewed the article by Hung and Plott, Information Cascades: Replication and an Extension To Majority Rule and Conformity-Rewarding Institutions, *American Economic Review,* 91, (2001): 1508 - 1520.[11] This article is also important not only because is provides additional confirmation of the integrity of Surowiecki's use of the literature but additionally, because it gives some sense of organizational characteristics which mediate the application of the principle. I recommend you also obtain a copy of that article. Skip all the mathematical notation, understand how the authors define "the individualistic institution", "the majority rule institution" and the "conformity rewarding institution" and then go directly to the conclusions section.

One usually has to handle the appeal to statistical justifications with some sensitivity toward the naïve members of the audience. There is often, if not always the person who will quote Twain (I think), "Lies, damned lies and statistics". This is the nature of the beast because the subject matter of statistical inference is the treatment of incomplete data. Surowiecki's work is on solid ground.

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[9] Surowiecki, P. 10.
[10] Surowiecki, P. 281.
[11] Ibid, p. 284